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Local geometry of the Fermi surface and magnetoacoustic response of two-dimensional electron systems in strong magnetic fields

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Abstract
A semiclassical theory for magnetotransport in a quantum Hall system near filling factor $\nu = 1/2$ based on the composite fermion physical picture is used to analyse the effect of local flattening of the composite fermion Fermi surface (CF-FS) upon magnetoacoustic oscillations. We report on calculations of the velocity shift and attenuation of a surface acoustic wave (SAW) which travels above the two-dimensional electron system, and we show that local geometry of the CF-FS could give rise to noticeable changes in the magnitude and phase of the oscillations. We predict these changes will be revealed in experiments, and will be used in further studies of the shape and symmetries of the CF-FS. The main conclusions reported here could be applied to analyse magnetotransport in quantum Hall systems at higher filling factors $\nu = 3/2$ and $5/2$ for a Fermi-liquid-like state of the system.

1. Introduction and background

A two-dimensional electron gas (2DEG) in a strong magnetic field reveals rich and complex physics. Near half filling of the lowest Landau level ($\nu = 1/2$) the ground state of such a system is shown to be a compressible Fermi-liquid-like state of composite fermions (CFs) [1]. These quasiparticles are distributed inside the composite fermion Fermi surface (CF-FS). A similar physical picture could be adopted to describe the 2DEG at half filling of the next Landau level ($\nu = 3/2$). Experimental evidence of the CF Fermi sea at $\nu = 1/2, 3/2$ was repeatedly obtained during the last decade [2].

For higher filling factors close to $\nu = N + 1/2$ where $N \geqslant 3$ is an integer, the exchange interaction would lead to an instability towards charge density wave formation in the relevant Landau levels. The ground state of the 2DEG for these filling factors corresponds to a charge density wave (CDW) and has a striped structure [3–6]. It could be described as a sequence of one-dimensional stripes alternating between the adjacent filling factors N and $N+1$. This gives rise to strikingly anisotropic transport properties of the 2DEG at half filling of higher Landau levels [7, 8] which were revealed in experiments [9–11].

The quantum Hall state at $\nu = 5/2$ is perhaps the most enigmatic due to its position in the magnetic field spectrum between the high Landau level $N \geqslant 3$ stripe phases and the low Landau level ($N \leqslant 1$) Fermi-liquid-like states. Theoretical studies of this state started from the model of paired CFs [12]. Depending on the interaction strength within the system, the 2DEG at $\nu = 5/2$ could reveal a striped state, a Fermi liquid or the paired state [13]. Numerical simulations presented in [13] give grounds to believe that at $\nu = 5/2$ the CF Fermi liquid undergoes condensation to the paired state at the low temperature limit. Also, there could be a transition from the Fermi-liquid to the striped phase when in-plane magnetic field is applied. Recently, experimental evidence of the CF-FS at $\nu = 5/2$ was obtained [14].

So, both theory and experiment give grounds to believe that the physical picture of CFs which form a Fermi sea could be successfully employed to describe the 2DEG states at half filling of the lowest Landau levels ($N \leqslant 2$). However, the geometry of the CF-FS has not been analysed up to the present. It is usually assumed that the CF Fermi liquid is isotropic, and the CF-FS is a circle in the two-dimensional quasimomentum space. This is an obvious oversimplification. Real samples commonly used in studies of the quantum Hall effect have 2DEGs deposited in GaAs/AlGaAs heterostructures. Therefore the crystalline field of the host semiconductor could significantly influence the CF-FS geometry distorting the original Fermi circle [15]. Another source of the CF-FS anisotropy, especially for higher filling factors ($\nu = 3/2, 5/2$), is the interaction in the electron system. The development of highly anisotropic charge-density wave formations (striped phases) at high filling factors including 5/2 gives us strong arguments to expect these interactions to work as an extra crystalline field in the Fermi-liquid state of the 2DEG at $\nu = 3/2, 5/2$. As a result the CF-FS shape could be further modified.

The theory of magnetotransport in metals shows that the FS local geometry noticeably affects the electron response of the metal to an external perturbation [16]. The change in the response occurs under the nonlocal regime of propagation of the disturbance when the mean free path of electrons l is large compared to the wavelength of the disturbance λ. The reason is that in this nonlocal regime only those electrons whose motion is somehow consistent with the propagating perturbation can strongly absorb its energy. These 'efficient' electrons are concentrated on small 'effective' segments of the FS.

When the FS includes flattened segments it leads to an enhancement of the contribution from these segments to the electron density of states (DOS) on the FS. Usually this enhanced contribution is small compared to the main term of the DOS which originates from all the remaining parts of the FS. Therefore it cannot produce noticeable changes in the response of the metal under the local regime of propagation of the disturbance ($l \ll \lambda$) when all segments of the FS contribute to the response functions essentially equally. However, the contribution to the DOS from the flattened section can be congruent to the contribution of a small 'effective' segment of the FS. In other words when the curvature of the FS becomes zero at some points on an 'effective' part of the FS it can give a significant enhancement of efficient electrons and, in consequence, a noticeable change in the response of the metal to the disturbance.

For the same reasons we can expect local geometrical features of the CF-FS to give significant effects on the 2DEG response to an external disturbance. As well as for conventional three-dimensional metals, these effects are to be revealed within a nonlocal regime ($l > \lambda$). It was shown before that the local flattening of the CF-FS could give rise to a strong anisotropy in the response of a 2DEG to a surface acoustic wave (SAW) [17]. Such an anomaly was observed in experiments on a modulated 2DEG near $\nu = 1/2$ [18].

Here, we analyse the influence of the CF-FS local geometry on so-called geometric resonances which were repeatedly observed in 2DEGs in strong magnetic field [2], as well as in conventional metals [19]. These oscillations could be revealed within a nonlocal regime, and they appear due to periodical reproduction of the most favourable conditions for the resonance absorption of the energy of the external disturbance by quasiparticles at stationary points on the cyclotron orbit where they move along the wavefront of the disturbance. When the external disturbance is associated with an acoustic wave these geometric resonances are also called magnetoacoustic oscillations [19]. In the following analysis we mostly consider magnetoacoustic oscillations in the 2DEG at $\nu = 1/2$ state, and we describe this state within the framework of Halperin–Lee–Read (HLR) theory [1]. However, we believe that the main results of the present analysis could be applied to study magnetotransport in 2DEGs at higher filling factors (3/2, 5/2) provided that the system is in a Fermi-liquid-like state.

2. Main equations

Due to the piezoelectric properties of GaAs, the velocity shift ($\Delta s/s$) and the attenuation rate (Γ) for the SAW propagating along the x axis across the surface of a heterostructure containing a 2DEG take the form [20]

$$\frac{\Delta s}{s} = \frac{\alpha^2}{2}\,\mathrm{Re}\left(1 + \frac{\mathrm{i}\sigma_{xx}}{\sigma_m}\right)^{-1}; \qquad \Gamma = -q\frac{\alpha^2}{2}\,\mathrm{Im}\left(1 + \frac{\mathrm{i}\sigma_{xx}}{\sigma_m}\right)^{-1}. \tag{1}$$

Here $\mathbf{q}$, $\omega = sq$ are the SAW wavevector and frequency, respectively, α is the piezoelectric coupling constant, $\sigma_m = \varepsilon s/2\pi$, ε is an effective dielectric constant of the background and σ_{xx} is the component of the electron conductivity tensor.

According to HLR theory, the electron resistivity tensor ρ at $\nu = 1/2$ is given by

$$\rho = \sigma^{-1} = \rho^{\mathrm{CF}} + \rho^{\mathrm{CS}} \tag{2}$$

where ρ^{CF} is the CF resistivity tensor, and the contribution ρ^{CS} originates from the Chern–Simons formulation of the theory. This tensor contains only off-diagonal elements $\rho_{xy}^{\mathrm{CS}} = -\rho_{yx}^{\mathrm{CS}} = 4\pi\hbar/e^2$.

The CF resistivity tensor is associated with the CF conductivity $\tilde{\sigma}$. As shown in the HLR paper [1], this tensor has the form

$$\rho^{\mathrm{CF}} = \tilde{\sigma}^{-1} + \frac{\mathrm{i}\omega(m - m_0)}{Ne^2} I \tag{3}$$

where I is the identity matrix of the second order, N is the electron density, m_0 is the CF bare band mass and m is their effective mass renormalized due to quasiparticle interactions. We carry out our analysis in a regime where $\rho_{xx}\rho_{yy} \ll \rho_{xy}^2$, therefore the relevant component of the electron conductivity could be written in the form

$$\sigma_{xx} = \frac{e^4}{(4\pi\hbar)^2}\left\{\frac{\tilde{\sigma}_{xx}}{\tilde{\sigma}_{xx}\tilde{\sigma}_{yy} + \tilde{\sigma}_{xy}^2} + \frac{\mathrm{i}\omega(m - m_0)}{Ne^2}\right\}. \tag{4}$$

Here, we concentrate on studies of magnetoacoustic oscillations in the 2DEG response. These oscillations are solely described by the first term in expression (4). The second term can noticeably contribute to the value of the electron conductivity but it does not change the characteristics of magnetoacoustic oscillations. On these grounds we omit the renormalization correction to the conductivity in further analysis. The CFs are supposed to experience not actual but reduced magnetic field $B_{\mathrm{eff}} = B - B_{1/2}$ where $B_{1/2}$ corresponds to one half filling of the lowest Landau level. Their motion could be described within a semiclassical approximation based on the Boltzmann transport equation. Following standard methods [21] we obtain

$$\tilde{\sigma}_{\alpha\beta} = \frac{2Nec}{B_{\mathrm{eff}}}\sum_n \frac{v_{n\beta}(-q)v_{n\alpha}(q)}{\mathrm{i}n - \omega/\Omega + 1/\Omega\tau}. \tag{5}$$

Here, Ω is the CF cyclotron frequency at the field B_{eff}; τ is the CF scattering time, and $v_{n\alpha}(q)$ are the Fourier transforms of the CF velocity components:

$$v_{nx}(q) = \frac{n}{2\pi}\frac{\Omega}{q}\int_0^{2\pi} \mathrm{d}\psi\ \exp\left\{ \mathrm{i}n\psi - \frac{\mathrm{i}q}{\Omega}\int_0^{\psi} v_x(\psi')\,\mathrm{d}\psi' \right\}; \tag{6}$$

$$v_{ny}(q) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\psi\ v_y(\psi)\exp\left\{ \mathrm{i}n\psi - \frac{\mathrm{i}q}{\Omega}\int_0^{\psi} v_x(\psi')\,\mathrm{d}\psi' \right\}. \tag{7}$$

The variable ψ included in these expressions is the angular coordinate of the CF cyclotron orbit.

The most favourable conditions for magnetoacoustic oscillations to be revealed occur at moderately strong effective magnetic field when $ql \gg \Omega\tau \gg 1$. Under these conditions the main contributions to the integrals over ψ in the expressions (6), (7) come from the neighbourhoods of stationary points at CF cyclotron orbits. As we show below, these contributions take the form determined with the local geometry of small effective segments of the CF-FS which correspond to the vicinities of the stationary points. When these segments are flattened, this leads to significant changes in the magnitude of the magnetoacoustic oscillations.

3. The CF-FS model

Within the commonly used isotropic model of the CF Fermi liquid at $\nu = 1/2$ the CF-FS is a circle, and its radius p_{F} equals $\sqrt{4\pi N\hbar^2}$ where N is the electron density. To develop a more realistic model of the CF-FS we include a periodic static electric field applied along the y direction which provides CFs with the potential energy of magnitude U_g ($\mathbf{g}$ is the wavevector of the electric field). The above electric field could originate from interactions with electrons of lower Landau levels at $\nu = 3/2, 5/2$ and from the crystalline field of the host semiconductor. The latter is especially important at $\nu = 1/2$. The point is that, wherever it comes from, this field distorts the CF-FS, including formation of local anomalies of the CF-FS curvature.

Assume for simplicity that the electric modulation is weak ($U_g \ll E_{\text{F}}$, where E_{F} is the CF Fermi energy). Then we can use the nearly-free-electron model to derive the energy–momentum relation for the CFs. When the modulation period is small enough $\hbar g > 2p_{\text{F}}$ we obtain

$$E(\mathbf{p}) = \frac{p_x^2}{2m} + \frac{p_y^{*2}}{2m} + \frac{(\hbar g)^2}{8m} - \sqrt{\left(\frac{\hbar g p_y^*}{2m}\right)^2 + U_g^2}. \tag{8}$$

Here $p_y^* = p_y - \hbar g/2$, m is the CF effective mass; U_g is the magnitude of the quasiparticle potential energy in the periodic electric field. Calculating the FS curvature,

$$K = \frac{1}{v^3}\left(2v_x v_y \frac{\partial v_x}{\partial p_y} - v_x^2 \frac{\partial v_y}{\partial p_y} - v_y^2 \frac{\partial v_x}{\partial p_x} \right) \tag{9}$$

with $\mathbf{v} = \partial E/\partial\mathbf{p}$, $v = \sqrt{v_x^2 + v_y^2}$, one can find it tending to zero when p_x tends to $\pm p_{\text{F}}(U_g/E_{\text{F}})^{1/2}$ (see figure 1).

In the vicinities of the corresponding points on the FS the quasiparticle velocities are nearly parallel to the y direction. Near these zero-curvature points we can derive an asymptotic expression for the energy–momentum relation (8). Introducing (p_{x_0}, p_{y_0}) by $p_{x_0} = \zeta p_{\text{F}}$, $p_{y_0} = p_{\text{F}}(1 - \frac{1}{\sqrt{2}}\zeta^2)$, where $\zeta = \sqrt{U_g/E_{\text{F}}}$, we can expand the variable p_y in powers of $(p_x - p_{x_0})$, and keep the lowest order terms in the expansion. We obtain

$$p_y - p_{y_0} = -\zeta(p_x - p_{x_0}) - \frac{2}{\zeta^4}\frac{(p_x - p_{x_0})^3}{p_{\text{F}}^2}. \tag{10}$$

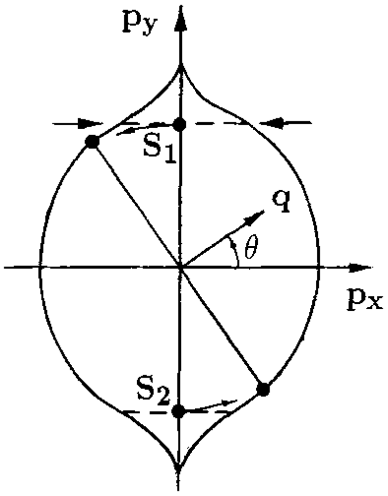

Figure 1. The shape of the CF-FS in the nearly-free-electron approximation (solid line), and the CF-FS described with equation (14) (dashed line). Points S_1 and S_2 are associated with the stationary points in the CF cyclotron orbit when the SAW wavevector **q** points in the p_x direction.

Near p_{x0}, where $(|p_x - p_{x_0}| < \zeta^2 p_F)$ the first term on the right side of equation (10) is small compared to the second one and can be omitted. So we have

$$E(\mathbf{p}) = \frac{4}{\zeta^4}\frac{p_F^2}{2m}\left(\frac{p_x - p_{x_0}}{p_F}\right)^3 + \frac{p_y^2}{2m}. \qquad (11)$$

The 'nearly-free'-particle model can be used when ζ^2 is very small. For larger U_g the local flattening of the CF-FS can be more significant. To analyse the contribution to the conductivity from these flattened parts we generalize the expression for $E(\mathbf{p})$ and define our dispersion as

$$E(\mathbf{p}) = \frac{p_0^2}{2m_1}\left|\frac{p_x}{p_0}\right|^\gamma + \frac{p_y^2}{2m_2}, \qquad (12)$$

where p_0 is a constant with the dimensions of momentum, m_i are the effective masses and γ is a dimensionless parameter which determines the shape of the CF-FS. When $\gamma > 2$ the CF-FS looks like an ellipse flattened near the vertices $(0, \pm\sqrt{m_2/m_1}p_0)$. Near these points the curvature is

$$K = -\frac{\gamma(\gamma-1)}{2p_0\sqrt{m_1/m_2}}\left|\frac{p_x}{p_0}\right|^{\gamma-2} \qquad (13)$$

and $K \to 0$ at $p_x \to 0$. The CF-FS will be flatter at $p_x = 0$ for larger parameter γ.

When $2p_F > \hbar g$ we have to consider the CF-FS as consisting of several branches belonging to several 'bands' or Brillouin zones. The modulating potential wavevector **g** in this case determines the size of the 'unit cell'. However, with this condition we also may expect some branches of the CF-FS to be flattened. Within an appropriate geometry of an experiment these flattened segments of the CF-FS become the effective parts of the FS. Consequently, the response of the CF system to the SAW could undergo significant changes. Prior to starting the analysis of these changes we remark that our model of the deformed CF-FS (12) could be easily generalized and accommodated to more complicated geometry of the electric field which determines the CF-FS shape and symmetries. However, even the simple model (12) captures the essential physics, enabling us to take local flattenings of the CF-FS into consideration. Therefore we adopt this model in the further analysis.

4. Results and discussion

When the SAW propagates along the x direction the vertices S_1, S_2 of the flattened ellipse (12) correspond to the stationary points on the CF cyclotron orbits (see figure 1). The enhanced DOS of quasiparticles in their vicinities influences the features of the magnetoacoustic oscillations. Using the stationary-phase method we obtain the following asymptotics for the Fourier transforms of the velocity components:

$$v_{nx}(q) = \frac{n\Omega}{q}\cos\left(qR - \frac{\pi n}{2} - \frac{\pi}{2\gamma}\right)X(qR); \tag{14}$$

$$v_{ny}(q) = -\mathrm{i}\sin\left(qR - \frac{\pi n}{2} - \frac{\pi}{2\gamma}\right)V(qR). \tag{15}$$

Here, $2R$ is the diameter of the CF cyclotron orbit in the direction of propagation of the SAW;

$$X(qR) = \frac{m^*}{p_0}V(qR) = \frac{2}{\pi}\left(\frac{m^*}{\sqrt{m_1 m_2}}\right)^{1/\gamma}\frac{\Gamma(1/\gamma)}{\gamma}\left(\frac{2}{qR}\right)^{1/\gamma} \equiv a\left(\frac{2}{qR}\right)^{1/\gamma} \tag{16}$$

where m^* is the CF cyclotron mass and $\Gamma(x)$ is the gamma function.

Using these results (16), as well as standard formulae [22],

$$\sum_{n=-\infty}^{\infty}\frac{1}{\omega + \mathrm{i}/\tau - n\Omega} = -\frac{\mathrm{i}\pi}{\Omega}\coth\frac{\pi(1-\mathrm{i}\omega\tau)}{\Omega\tau}; \tag{17}$$

$$\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{\omega + \mathrm{i}/\tau - n\Omega} = -\frac{\mathrm{i}\pi}{\Omega}\frac{1}{\sinh\left[\pi(1-\mathrm{i}\omega\tau)/\Omega\tau\right]} \tag{18}$$

we can transform the expressions (5) for the CF conductivity components to the form

$$\tilde{\sigma}_{xx} = \frac{2}{\rho_0}\frac{(1-\mathrm{i}\omega\tau)}{(ql)^2}; \tag{19}$$

$$\tilde{\sigma}_{xy} = -\sigma_{yx} = -\frac{2}{\rho_0}\frac{g^2}{(ql)^2}\left(\frac{qR}{2}\right)^{1-2/\gamma}\frac{(1-\mathrm{i}\omega\tau)\sin\left(2qR - \pi/\gamma\right)}{\sinh\left[\pi(1-\mathrm{i}\omega\tau)/\Omega\tau\right]}; \tag{20}$$

$$\tilde{\sigma}_{yy} = \frac{2}{\rho_0}\frac{d^2}{ql}\left(\frac{qR}{2}\right)^{1-2/\gamma}\left\{\coth\frac{\pi(1-\mathrm{i}\omega\tau)}{\Omega\tau} - \cos\left(2qR - \frac{\pi}{\gamma}\right)\sinh^{-1}\frac{\pi(1-\mathrm{i}\omega\tau)}{\Omega\tau}\right\} \tag{21}$$

where $\rho_0 = m^*/Ne^2\tau$ is the CF Drude resistivity; $l = \frac{\tau}{m^*}\sqrt{\frac{A}{\pi}}$ and A is the area of the CF-FS. The factors d^2 and g^2 included in (20) and (21) are the dimensionless constants of the order of unity:

$$g^2 = \pi a^2\sqrt{\frac{m_1}{m_2}}; \qquad d^2 = \frac{(\pi a)^2}{\sqrt{\pi A}}\sqrt{\frac{m_1}{m_2}}p_0. \tag{22}$$

For a circular CF-FS we have $\gamma = 2$, $p_0 = p_{\mathrm{F}}$, $g^2 = d^2 = 1$, and our expressions (20) and (21) take on the form

$$\tilde{\sigma}_{xy} = -\tilde{\sigma}_{yx} = \frac{2}{\rho_0}\frac{1-\mathrm{i}\omega\tau}{(ql)^2}\cos(2qR)\sinh^{-1}\left[\frac{\pi(1-\mathrm{i}\omega\tau)}{\Omega\tau}\right]; \tag{23}$$

$$\tilde{\sigma}_{yy} = \frac{2}{\rho_0}\frac{1}{ql}\left\{\coth\left[\frac{\pi(1-\mathrm{i}\omega\tau)}{\Omega\tau}\right] - \sin(2qR)\sinh^{-1}\left[\frac{\pi(1-\mathrm{i}\omega\tau)}{\Omega\tau}\right]\right\}. \tag{24}$$

Comparison of our expressions (19)–(21) with the results for a Fermi circle shows that the local flattening of the CF-FS near the points which correspond to the stationary points of the CFs cyclotron orbit enhances the magnitude of the magnetoacoustic oscillations of the CF conductivities. A similar effect was studied before for conventional metals [23]. The effect originates from the enhancement of the quasiparticle DOS at flattened segments of the FS.

The above considered enhancement of magnetoacoustic geometric oscillations could be manifested only when the stationary points on the CF cyclotron orbit correspond to the points located at flattened segments of the CF-FS. Therefore, the effect has to be very sensitive to variations in the direction of the SAW propagation. Suppose that the SAW travels at some angle θ with respect to the symmetry axis of the CF-FS as shown in figure 1. Then the stationary points slip from the flattened pieces and fall into 'normal' segments of the FS whose curvature takes on nonzero values. Due to the lower DOS of quasiparticles at these 'normal' CF-FS segments, the number of efficient CFs which can participate in the absorption of the SAW energy decreases when the angle θ increases. This results in the noticeable reduction of the oscillations.

Assuming a nonzero value for the angle θ, we can present Fourier transforms of the CF velocity components in the form

$$v_{nx}(q) = \frac{n\Omega}{q}\left[\cos\left(qR - \frac{\pi n}{2}\right)S_\gamma(qR,\theta) + \sin\left(qR - \frac{\pi n}{2}\right)W_\gamma(qR,\theta)\right]; \tag{25}$$

$$v_{ny}(q) = -\frac{\mathrm{i}p_0}{m^*}\left[\sin\left(qR - \frac{\pi n}{2}\right)S_\gamma(qR,\theta) - \cos\left(qR - \frac{\pi n}{2}\right)W_\gamma(qR,\theta)\right]. \tag{26}$$

Here,

$$S_\gamma(qR,\theta) = \frac{2}{\pi}\left(\frac{m^*}{\sqrt{m_1 m_2}}\right)^{1/\gamma}\int_0^\infty \mathrm{d}y\,\cos\left[\frac{qR}{2}\left(\frac{m_2}{m_1}\sin^2\theta y^2 + \cos^\gamma\theta y^\gamma\right)\right]; \tag{27}$$

$$W_\gamma(qR,\theta) = \frac{2}{\pi}\left(\frac{m^*}{\sqrt{m_1 m_2}}\right)^{1/\gamma}\int_0^\infty \mathrm{d}y\,\sin\left[\frac{qR}{2}\left(\frac{m_2}{m_1}\sin^2\theta y^2 + \cos^\gamma\theta y^\gamma\right)\right]. \tag{28}$$

Now, we expand these functions S_γ in series in powers of a dimensionless parameter $\xi(\xi = \frac{m_2}{m_1}(\frac{qR}{2})^{1-2/\gamma}\tan^2\theta)$. For small angles θ we have $\xi \ll 1$, and the expansion takes the form [22]

$$S_\gamma(qR,\theta) = \frac{1}{\gamma\cos\theta}\left(\frac{2}{qR}\right)^{1/\gamma}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!}\xi^r\Gamma\left(\frac{2r+1}{\gamma}\right)\cos\left[\pi\frac{1-r(\gamma-2)}{2\gamma}\right]. \tag{29}$$

As θ increases to the values guaranteeing the inequality $\xi > 1$ to be valid, we have to use a different power expansion, namely

$$S_\gamma(qR,\theta) = \frac{1}{2\sin\theta}\sqrt{\frac{m_1}{m_2}\frac{2}{qR}}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!}\xi^{-\gamma r/2}\Gamma\left(\frac{\gamma r+1}{2}\right)\cos\left[\frac{\pi}{4}(r(\gamma-2)+1)\right]. \tag{30}$$

Expansions for the function $W_\gamma(qR,\theta)$ could be obtained from (29) and (30) by replacing cosines by sines of the same angles. When $\theta \to 0$ we arrive at our former asymptotic expressions for the CF velocity components (14) and (15). While θ increases, the functions $S_\gamma(qR,\theta)$ and $W_\gamma(qR,\theta)$ diminish and approach the value $(\pi m_1/4m_2 qR)^{1/2}$ which is a typical estimate for an elliptical CF-FS as $\theta \to 90°$.

Differences in magnitudes of geometric oscillations of the CF conductivity components are manifested in the electron conductivity. Substituting our results for $\tilde{\sigma}_{\alpha\beta}$ into (4) we have

$$\sigma_{xx} = \frac{qe^2}{8\eta^2 p_0}[qRF^2(qR)]^{-1}\frac{\sinh[\pi(1-\mathrm{i}\omega\tau)/\Omega\tau]}{\cosh[\pi(1-\mathrm{i}\omega\tau)/\Omega\tau] - \cos(2qR - 2\Phi)}. \tag{31}$$

Here,

$$\eta^2 = 2\sqrt{\frac{A}{\pi p_0^2}\frac{m_2}{m_1}}; \qquad \tan\Phi = \frac{W_\gamma(qR,\theta)}{S_\gamma(qR,\theta)}. \tag{32}$$

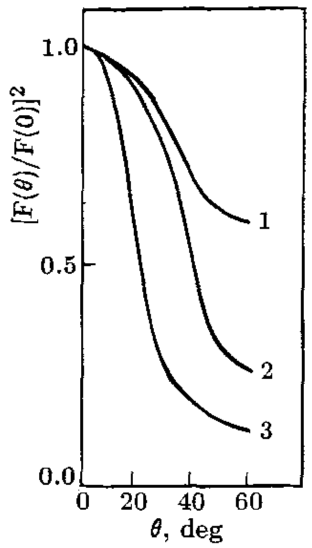

Figure 2. Angular dependence of the amplitude of magnetoacoustic oscillations. The curves are plotted for $qR = 10$; $m_1 = m_2$; $\gamma = 4$ (curve 1); $\gamma = 6$ (curve 2); and $\gamma = 8$ (curve 3).

When $\theta = 0$, and the locations of the stationary points correspond to flattened segments of the CF-FS, the expression for σ_{xx} takes on the form

$$\sigma_{xx} = \frac{qe^2}{8d^2 p_0}\left(\frac{2}{qR}\right)^{1-2/\gamma} \frac{\sinh[\pi(1-\mathrm{i}\omega\tau)/\Omega\tau]}{\cosh[\pi(1-\mathrm{i}\omega\tau)/\Omega\tau] - \cos(2qR - \pi/\gamma)}. \quad (33)$$

We compare this expression with the corresponding result for a circular CF-FS

$$\sigma_{xx} = \frac{qe^2}{8p_F} \frac{\sinh[\pi(1-\mathrm{i}\omega\tau)/\Omega\tau]}{\cosh[\pi(1-\mathrm{i}\omega\tau)/\Omega\tau] - \sin(2qR)}, \quad (34)$$

and we see that both amplitude and phase of the geometric oscillations in the electron conductivity differ from those for the CF Fermi circle. The same could be applied to magnetoacoustic oscillations described with the expressions (1) and (2). The angular dependence of the amplitude factor of magnetoacoustic oscillations $F_\gamma^2(qR,\theta) = S_\gamma^2(qR,\theta) + W_\gamma^2(qR,\theta)$ is presented in figure 2. We see in this figure that the effect of local flattening of the CF-FS on the oscillation amplitude remains distinguishable even for moderate flattenings ($\gamma = 4$).

The present analysis was carried out assuming isotropic scattering in the system. Anisotropy in scattering may significantly affect magnetotransport characteristics of the 2DEG [24]. As a result, magnitudes of magnetoacoustic oscillations could be noticeably reduced compared to those obtained adopting a simple model of isotropic scattering. This brings difficulties in quantitative comparison of our results with experimental data. However, it does not change our main conclusion, namely that oscillation magnitudes strongly depend on the local geometry of the CF-FS. When the effective parts of the CF-FS are flattened, the amplitude of the magnetoacoustic oscillations drops. We also conclude that varying the direction of propagation of the SAW we can observe angular dependence of the oscillation amplitude. The latter originates from the angular dependence of the CF conductivities discussed before. We have grounds to expect it to be revealed in experiments.

The very essence of the adopted model (12) is that it describes a curve whose curvature turns zero at $p_x = 0$ provided that the curve has symmetries of an ellipse. The expression for the curvature (13) shows that any such curve takes the form (12) with $\gamma > 2$ near $p_x = 0$. Otherwise, it cannot be flattened at these points. Within this approach we treat γ as a phenomenological parameter. Actual values of γ could be discovered in experiments where the FS local geometry is revealed. This is the only trustworthy way to estimate the above parameter. First principles calculations are not accurate enough to produce reliable results on such fine geometrical features of the FSs as local flattenings. Up to the present, only a few relevant experiments have been carried out, so it is impossible to give a proper and realistic estimation for this parameter γ. Therefore, merely preliminary remarks concerning possible value of γ are presented here. It was shown that the anomaly in the response of the modulated 2DEG at $\nu = 1/2$ to the SAW [18] could originate from the distortion of the CF-FS with the modulating field. The anisotropic response was observed at modulation rates $\Delta N/N \sim 0.01$–0.05. The modulated 2DEG could be treated as some kind of striped structure artificially created with the applied electric field. The characteristic energy E_{A} associated with this field induced anisotropy could be roughly estimated as $E_{\mathrm{A}} \sim \frac{\Delta N}{N} E_{\mathrm{F}}$. The sample used in the experiments of [18] had electron density $N \approx 0.7 \times 10^{11}$ cm^{-2} which gives $E_{\mathrm{A}} \sim 10$–50 mK. Theoretical analysis of [17] based on the model of the deformed CF-FS gives a reasonably good approximation for the magnitude of the anomalous peak in the SAW velocity shift at $\gamma \sim 3$–6. Hartree–Fock calculations of the anisotropy energy in a quantum Hall striped state at $\nu = 5/2$ evaluated this energy as 30.00 mK per electron assuming electron density 2.7×10^{11} cm^{-2} [25]. For the Fermi-liquid-like state of this system E_{F} could be estimated as 750 mK. We keep in mind that a native anisotropy associated with the Fermi-liquid-like state at $\nu = 5/2$ is probably much weaker than estimated above. Nevertheless, we may expect the ratio $E_{\mathrm{A}}/E_{\mathrm{F}}$ to take on values of the order of 0.05 or even greater. In this case the parameter γ could accept values of the order of 10, and the effect of the CF-FS local geometry on the magnetoacoustic oscillations would be quite distinguishable.

A significant anisotropy in a 2DEG response to a SAW is revealed when an external static electric periodical modulation is applied to the system. The effect was repeatedly analysed before assuming an isotropic density of the 2DEG states (see e.g. [24] and references therein). The anisotropy arises due to the break in the symmetry of the system produced by the modulation, and it depends on the mutual orientation of the modulation equipotential lines and the SAW wavevector. This effect is distinct from the anisotropy in the 2DEG response originating from the inherent FS local geometry which is studied in the present work. However, these different effects could be related, for one can treat an external modulation as a field, distorting the originally circular FS to a more complicated shape including local flattenings.

Finally, we believe that the Fermi-liquid state of a 2DEG in the quantum Hall regime at $\nu = 1/2, 3/2, 5/2$ is anisotropic and exhibits an anisotropic CF-FS. The CF-FS geometry reflects symmetries of the crystalline fields of the host semiconductor. For higher filling factors $\nu = 3/2$ and 5/2 it also could show effects of interactions in the electron system. It has already been found that screening due to polarization of remote Landau levels plays an essential role for the preferred orientation of the stripes induced by an in-plane magnetic field at $\nu = 5/2$ [25]. Therefore, we may conjecture that at weaker in-plane fields when the Fermi-liquid like state of the 2DEG still exists these polarization effects could give extra anisotropies to the CF-FS.

Anisotropic CF-FSs usually include some flattened segments. Even a naive model (8) based on the nearly-free-electron approach demonstrates that local flattening of the CF-FS appears as a result of a weak deformation of the latter with an electric modulation. In general, local flattenings are initiated with electric fields acting within a 2DEG like crystalline fields in usual metals. Accordingly, locations of the flattened segments conform with the symmetries of the CF-FS and could reveal these symmetries. The results of the present analysis show that magnetoacoustic oscillations in the velocity shift and attenuation of the SAW travelling in piezoelectric GaAs/AlGaAs heterostructures above the 2DEG could be used as a tool to discover local flattenings at the CF-FS when the 2DEG is in the quantum Hall regime in the Fermi-liquid-like state. This could give a new knowledge of the shape and symmetries of the CF-FS and, consequently, a better understanding of the magnetotransport in quantum Hall systems near half filling of the lowest Landau levels. It would be especially interesting to compare symmetries of the CF-FS in the Fermi-liquid-like state of the 2DEG at $\nu = 5/2$ with the characteristic symmetries of the striped state of the system at the same filling factor. It is possible that such a comparison would give some unusual results, providing a new insight into the nature of the transition from the Fermi-liquid to the striped phase of the 2D electron system.

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